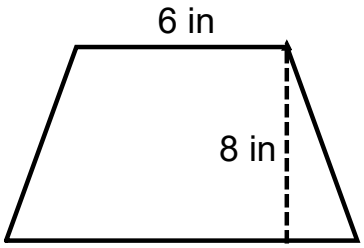
4.6 The Trapezoidal Rule

Objective: You will be able to:

- approximate a definite integral using the Trapezoidal Rule

Warm Up

Find the area of the trapezoid:



$$\frac{(10+6)}{2}(8)$$

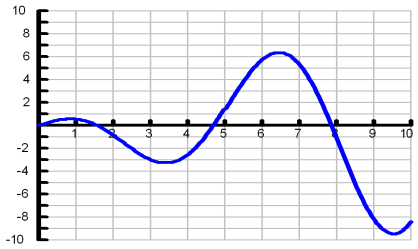
$64\ in^2$

Why Approximate Integration?

- Can't always find an antiderivative

Example: $\int_0^1 e^{x^2}\,dx$

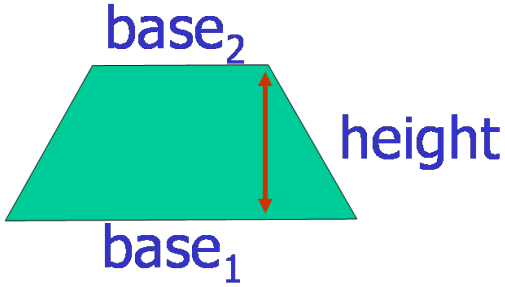
- Don't always know the function



Definition

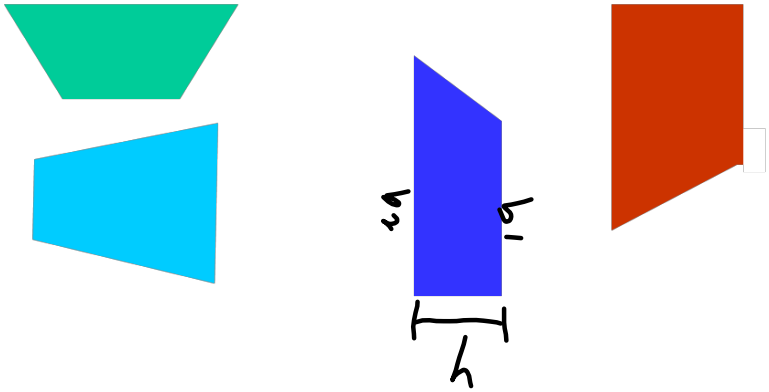
Area of a Trapezoid

$$\text{Area} = h\left(\frac{\text{base}_1 + \text{base}_2}{2}\right)$$



Important Idea

Trapezoids come in all shapes and sizes...



Important Idea

Area under a curve can also be defined as the sum of the area of the trapezoids under the curve.

Calculus in Motion:
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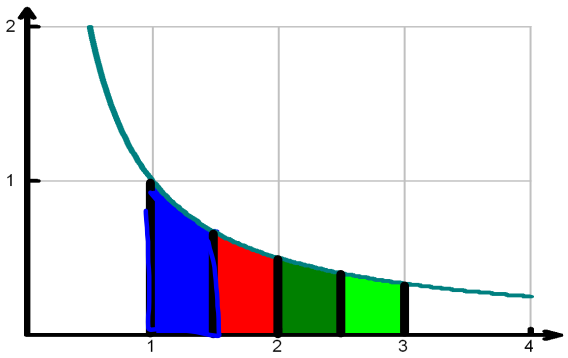


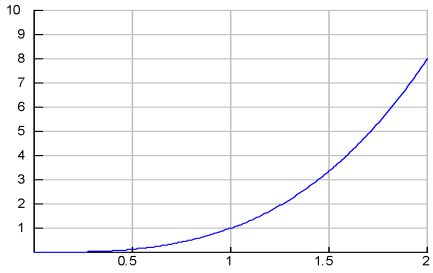
$$\int_a^b f(x)dx \approx Trapezoid_n = \frac{1}{2}\bullet\frac{b-a}{n}\bullet(L_n + R_n)$$

Trapezoidal Rule:

$$= \frac{b-a}{2n}[f(x_0) + 2f(x_1) + \ldots + 2f(x_{n-1}) + f(x_n)]$$

Use the Trapezoidal approximation to approximate the value of the definite integral for n = 4.
Compare your answer with the exact value of the definite integral.

$$\int_0^2 x^3\,dx$$

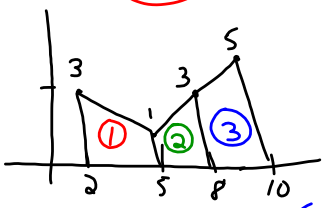


5. The table below gives data points for a continuous function f on [2, 10].

x	2	5	~~7~~	8	10
$f(x)$	3	1	~~5~~	3	5

Using subdivisions [2, 5], [5, 8] and [8, 10], what is the trapezoid approximation of $\int_2^{10} f(x)\,dx$?

(A) 18 (B) 20 (C) 22 (D) 24 (E) 40



B

$\frac{(1+3)}{2}\cdot 3 + \frac{(3+1)}{2}\cdot 3 + \frac{(5+3)}{2}(2)$

$\frac{1}{2}(4\cdot 3 + 4\cdot 3 + 8\cdot 2)$

$\frac{1}{2}(12+12+16)$

$\frac{1}{2}(40)$

20

12
12
16
40

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Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

$\frac{55-62}{8-6} = \frac{-7}{2}$

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

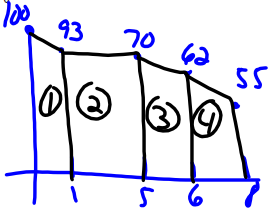
$-7/2$ °C/cm

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

b)



$\frac{(100+93)}{2}(1) + \frac{(93+70)}{2}(4) + \frac{(70+62)}{2}(1) + \frac{(62+55)}{2}(2)$

Attachments

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