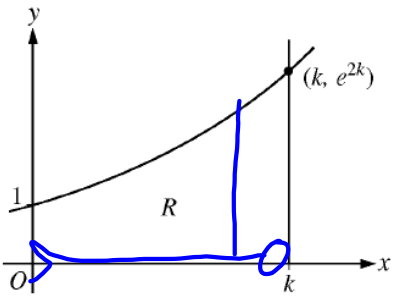
2011 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

$\pi\int_0^k (e^{2x})^2\,dx$



3. Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of f, the coordinate axes, and the vertical line $x = k$, where $k > 0$. The region R is shown in the figure above.

(b) The region R is rotated about the x-axis to form a solid. Find the volume, V, of the solid in terms of k.

(b) Volume $= \pi\int_0^k \left(e^{2x}\right)^2 dx = \pi\int_0^k e^{4x}\,dx = \frac{\pi}{4}e^{4x}\Big|_{x=0}^{x=k} = \frac{\pi}{4}e^{4k} - \frac{\pi}{4}$

$\frac{\pi}{4}(e^{4k}-1)$

4 : { 1 : integrand; 1 : limits; 1 : antiderivative; 1 : answer }

$u = 4x$
$du = 4\,dx$
$\frac{1}{4}du = dx$

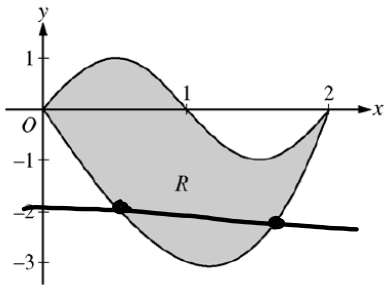
$\frac{\pi}{4}\int e^u\,du$

$\frac{\pi}{4}e^{4x}\Big]_0^k$

2008 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.
   (a) Find the area of R.
   (b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

(a) $\sin(\pi x) = x^3 - 4x$ at $x = 0$ and $x = 2$

Area $= \int_0^2 \left(\sin(\pi x) - \left(x^3 - 4x\right)\right) dx = 4$

3 : { 1 : limits; 1 : integrand; 1 : answer }

(b) $x^3 - 4x = -2$ at $r = 0.5391889$ and $s = 1.6751309$

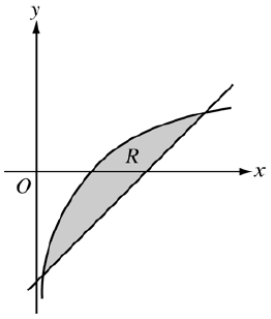
The area of the stated region is $\int_r^s \left(-2 - \left(x^3 - 4x\right)\right) dx$

2 : { 1 : limits; 1 : integrand }

2006 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.
   (a) Find the area of R.
   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

$\ln(x) = x - 2$ when $x = 0.15859$ and 3.14619.
Let $S = 0.15859$ and $T = 3.14619$

(a) Area of $R = \int_S^T (\ln(x) - (x-2))\,dx = 1.949$

3 : { 1 : integrand; 1 : limits; 1 : answer }

(b) Volume $= \pi \int_S^T \left((\ln(x)+3)^2 - (x-2+3)^2\right) dx$
$= 34.198$ or 34.199

3 : { 2 : integrand; 1 : limits, constant, and answer }

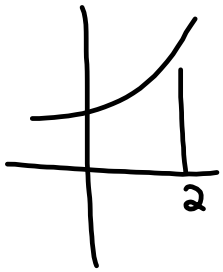
80. Let R be the region enclosed by the graph of $y = 1 + \ln\left(\cos^4 x\right)$, the x-axis, and the lines $x = -\frac{2}{3}$ and $x = \frac{2}{3}$. The closest integer approximation of the area of R is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

80. B The area is given by $\int_{-\frac{2}{3}}^{\frac{2}{3}} \left(1 + \ln(\cos^4 x)\right) dx = 0.919$

10. What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

(A) $2e - 2$ (B) $2e$ (C) $\frac{e}{2} - 1$ (D) $\frac{e-1}{2}$ (E) $e - 1$



$\int_0^2 e^{x/2}\,dx$ 2012 ab A

$u = x/2$

$du = 1/2\,dx$

$2\,du = dx$

$2\int_0^2 e^u\,du$

$2e^{x/2}\Big]_0^2$

89. What is the volume of the solid generated when the region bounded by the graph of $x = \sqrt{y-2}$ and the lines $x = 0$ and $y = 5$ is revolved about the y-axis?

(A) 3.464 (B) 4.500 (C) 7.854 (D) 10.883 (E) 14.137

$x^2 = y-2$

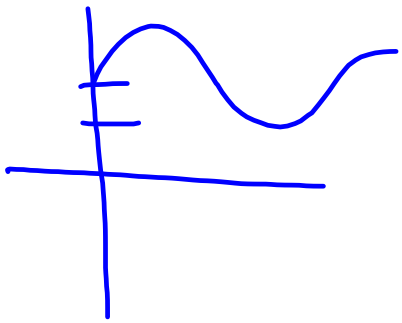
$x^2+2=y$

$$\pi \int_0^5 (\sqrt{y-2})^2 \, dy$$

2013 ab E

79. A vase has the shape obtained by revolving the curve $y = 2 + \sin x$ from $x = 0$ to $x = 5$ about the x-axis, where x and y are measured in inches. What is the volume, in cubic inches, of the vase?

(A) 10.716 (B) 25.501 (C) 33.666 (D) 71.113 (E) 80.115



$$\pi \int_0^5 (2+\sin x)^2 \, dx$$

2014 ab E