Chapter 9
Infinite Series

The importance of understanding the infinitestimals...

-How much is a drop of lemonade?
-A drop I'll give you for nothin'!

-Can I get a cupful of drops?

9.1 Sequences

Usually sequences and series are thought of together

Sequence: a **listing** of numbers

Ex. 1

1, 2, 3, 4, ...

goes off to ∞

(Diverges)

Ex. 2

1, .1, .01, .001, .0001, ...

$\longrightarrow 0$

(Converges)

The thing you will want to ask yourself:

Does the sequence converge?

Monotonic:

1) increasing
2) decreasing

Oscillate:

0, 1, 0, -1, 0, 1, 0, -1, ...

Ex. 2

1, .1, .01, .001, .0001, ...

1st term	2nd term	3rd term	4th term
1	2	3	4
a_1	a_2	a_3	a_4
1	.1	.01	.001

Typically you are given a formula

$$a_n = \frac{3(-1)^n}{n!}$$

In case you forgot... 5 factorial

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$a_n = \frac{3(-1)^n}{n!}$$

by Definition $0! = 1$

$$a_1 = \frac{3(-1)^1}{1!} \qquad a_2 = \frac{3(-1)^2}{2!} \qquad a_3 = \frac{3(-1)^3}{3!}$$

$3(-1)$ $\qquad$ $\frac{3(1)}{2}$ $\qquad$ $\frac{\cancel{3}(-1)}{\cancel{3}\cdot 2\cdot 1}$

-3 $\qquad$ $3/2$ $\qquad$ $-1/2$

For a particular sequence, does it converge or diverge?

We won't cover every case, but this is what you'll do.

Ex. 1

$$a_n = n(n-1)$$

If you have no feeling, just start plugging in numbers. It might give some intuition of what is going on. Best mathematicians do this to get a feel of what's going on.

a_1 $1(1-1)$ 0

a_2 $2(2-1)$ 2

a_3 $3(3-1)$ 6

a_4 $4(4-1)$ 12

Diverge

Ex. 1

$$a_n = n(n-1)$$

n is going to be getting bigger and bigger

$$\lim_{n\to\infty} n(n-1) = \infty$$

$\infty(\infty)$ diverges

Ex. 2

$$\lim_{n\to\infty} \frac{2^n}{3^{n+1}}$$

Could use L'Hopitals, denominator getting bigger much faster than numerator

but...
could also rewrite

$$\lim_{n\to\infty} \frac{2^n}{3^{n+1}} = \lim_{n\to\infty} \frac{2^n}{3^n \cdot 3^1}$$

$$\frac{1}{3}\lim_{n\to\infty}\left(\frac{2}{3}\right)^n$$

converges if number inside does: $-1 \leq \text{some number} \leq 1$

Ex. 3

$$\left\{\frac{(2n-1)!}{(2n+1)!}\right\}$$ notation same as $$a_n = \frac{(2n-1)!}{(2n+1)!}$$

$$\lim_{n\to\infty}\frac{(2n-1)!}{(2n+1)!}$$ try to simplify...

$$\frac{(2n-1)!}{(2n+1)(2n)(2n-1)!}$$

$$\lim_{n\to\infty}\frac{1}{(2n+1)(2n)}$$

$$0$$

Ex. 4

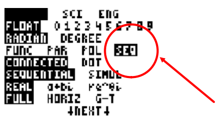
$$a_n = \frac{\cos^2 n}{2^n}$$

$$\lim_{n\to\infty}\frac{\cos^2 n}{2^n}$$

$$\frac{(\cos n)^2}{2^n} \qquad \frac{0 \le n \le 1}{2^n}$$

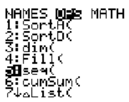
Bottom Heavy 0

Enter a sequence in your calculator

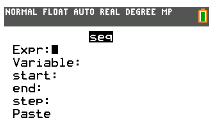


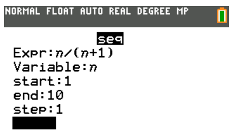
$$a_n = \frac{n}{(n+1)}$$

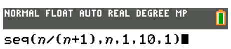
(next slide graphing)

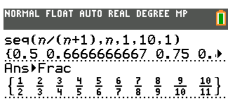


2nd Stat (List), arrow over to ops



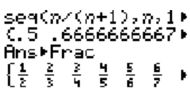




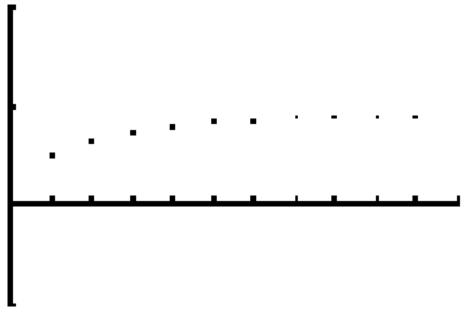


seq(n/(n+1),n,1-10)

math frac



```
Plot1 Plot2 Plot3
nMin=1
·u(n)=█/(n+1)
 u(nMin)=
·v(n)=
 v(nMin)=
·w(n)=
 w(nMin)=
```



```
WINDOW
 nMin=1
 nMax=10
 PlotStart=1
 PlotStep=1
 Xmin=0
 Xmax=11
↓Xscl=1
```

```
WINDOW
↑PlotStep=1
 Xmin=0
 Xmax=11
 Xscl=1
 Ymin=-1
 Ymax=2
 Yscl=█
```

1,1,2,3,5, 8, 13, 21, 34

Fibonacci Sequence

most famous **recursive** sequence

In Exercises 1–10, write the first five terms of the sequence.

2. $a_n = \dfrac{3^n}{n!}$

3, 4.5, 4.5, 3.375, 2.025

8. $a_n = (-1)^{n+1}\left(\dfrac{2}{n}\right)$

$2, -1, \frac{2}{3}, -\frac{1}{2}, \frac{2}{5}$

In Exercises 11–14, write the first five terms of the recursively defined sequence.

11. $a_1 = 3,\ a_{k+1} = 2(a_k - 1)$

$a_1 = 3$

$a_{1+1} = a_2 = 2(3-1) = 4$

$a_{2+1} = a_3 = 2(4-1) = 6$

$a_{3+1} = a_4 = 2(6-1) = 10$

$a_{4+1} = a_5 = 2(10-1) = 18$