Ex. 4

$\boxed{\text{break into 2 parts: } c=0 \text{ is a convenient value}}$

$$\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}}\,dx = \int_{-\infty}^{0} + \int_{0}^{\infty}$$

$$\lim_{a\to-\infty}\int_a^0 \frac{e^x}{1+e^{2x}}\,dx + \lim_{b\to\infty}\int_0^b \frac{e^x}{1+e^{2x}}\,dx$$

$a^2=1 \quad u^2=e^{2x}$

$a=1 \quad u=(e^{2x})^{1/2}$

$u=e^x$

$du=e^x dx$

$\frac{1}{1}\arctan\frac{e^x}{1}$

$$\lim_{a\to-\infty}\left[\arctan e^x\right]_a^0 + \lim_{b\to\infty}\left[\arctan e^x\right]_0^b$$

$$\lim_{a\to-\infty}\left\{\begin{matrix}F(0)-F(a)\\ \arctan e^0 - \arctan e^a\end{matrix}\right\} + \lim_{b\to\infty}\left[\begin{matrix}F(b)-F(0)\\ \arctan e^b - \arctan e^0\end{matrix}\right]$$

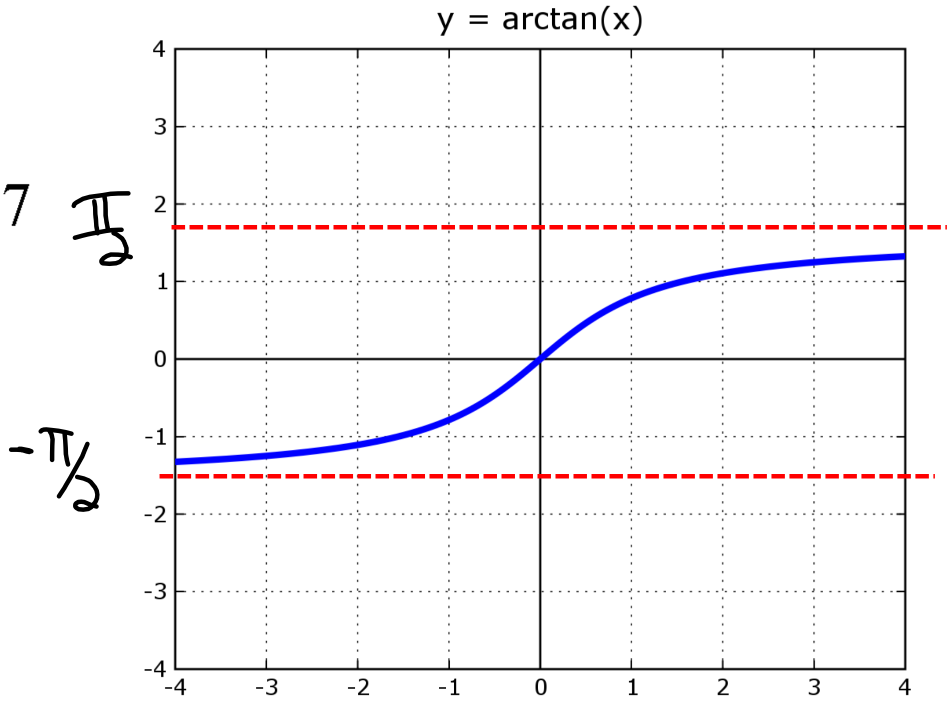
$\arctan 1 - \arctan 0 \quad + \frac{\pi}{2} - \arctan 1$

$\left[\frac{\pi}{4} - 0\right] + \left[\frac{\pi}{2} - \frac{\pi}{4}\right]$

$\frac{\pi}{2}$

Function	*Domain*	*Range*
arcsin $\sin^{-1}$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
arccos $\cos^{-1}$	$-1 \le x \le 1$	$0 \le y \le \pi$
arctan $\tan^{-1}$	$-\infty \le x \le \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

$$\frac{\pi}{2} \approx 1.57$$



Infinite Discontinuities

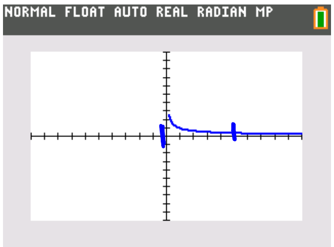
at or between limits

DEFINITION OF IMPROPER INTEGRALS WITH INFINITE DISCONTINUITIES

1. If f is continuous on the interval $[a, b)$ and has an infinite discontinuity at b, then
$$\int_a^b f(x)\,dx = \lim_{c\to b^-} \int_a^c f(x)\,dx.$$
2. If f is continuous on the interval $(a, b]$ and has an infinite discontinuity at a, then
$$\int_a^b f(x)\,dx = \lim_{c\to a^+} \int_c^b f(x)\,dx.$$
3. If f is continuous on the interval $[a, b]$, except for some c in (a, b) at which f has an infinite discontinuity, then
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$

In the first two cases, the improper integral **converges** if the limit exists—otherwise, the improper integral **diverges.** In the third case, the improper integral on the left diverges if either of the improper integrals on the right diverges.

Ex. 5
$$\int_0^5 \frac{dx}{\sqrt{x}}$$



$$\lim_{a\to 0^+} \int_a^5 x^{-1/2}\,dx$$

$$\lim_{a\to 0^+} \left[2x^{\frac{1}{2}}\right]_a^5$$

$$\lim_{a\to 0^+} F(5) - F(a)$$
$$2(5)^{1/2} - 2(a)^{1/2}$$

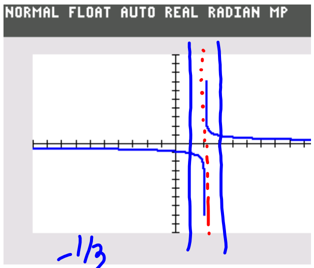
$$2\sqrt{5} - 0$$

$$2\sqrt{5}$$

Ex. 6

be careful

$$\int_1^3 \frac{1}{(x-2)^{1/3}}\,dx$$



$$\lim_{b\to 2^-}\int_1^b (x-2)^{-1/3}\,dx + \lim_{a\to 2^+}\int_a^3 (x-2)^{-1/3}\,dx$$

$u = x-2$

$du = dx$

$\int u^{-1/3}dx$

$$\lim_{b\to 2^-}\left[\frac{3}{2}(x-2)^{2/3}\right]_1^b + \lim_{a\to 2^+}\left[\frac{3}{2}(x-2)^{2/3}\right]_a^3$$

$$F(b)-F(1) \quad + \quad F(3)-F(a)$$

$$\frac{3}{2}\left[0-1\right] \quad + \frac{3}{2}\left[1-0\right]$$

$$-\frac{3}{2} \quad + \frac{3}{2}$$

$$0$$

Ex. 7

(ex. 8 p. 582)

your turn

$$\int_{-1}^2 \frac{dx}{x^3}$$

$$\lim_{b\to 0^-}\int_{-1}^b x^{-3}\,dx + \lim_{a\to 0^+}\int_a^2 x^{-3}\,dx$$

$$\lim_{b\to 0^-}\left[\frac{x^{-2}}{-2}\right]_{-1}^b$$

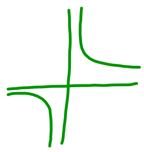
$$-\frac{1}{2}\lim_{b\to 0^-}\left[\frac{1}{x^2}\right]_{-1}^b$$

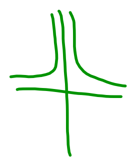
$$F(b)-F(-1)$$

$$-\frac{1}{2}\lim_{b\to 0^-}\left[\frac{1}{b^2} - \frac{1}{(-1)^2}\right]$$

$$\infty$$

Diverges

$\frac{1}{x} =$ 

$\frac{1}{x^2} =$ 

25. $\int_1^\infty xe^{-x^2}\,dx$ is

(A) $-\frac{1}{e}$ (B) $\frac{1}{2e}$ (C) $\frac{1}{e}$ (D) $\frac{2}{e}$ (E) divergent

25) B

$$\lim_{b\to\infty}\int_1^b xe^{-x^2}\,dx$$

$u=-x^2$
$du=-2x\,dx$
$-\frac{1}{2}du=x\,dx$

$$-\frac{1}{2}\lim_{b\to\infty}\int_1^b e^u\,du$$

$$-\frac{1}{2}\lim_{b\to\infty}\left[\overset{e^u}{e^{-x^2}}\right]_1^b$$

$$-\frac{1}{2}\lim_{b\to\infty}\left[e^{-b^2}-e^{-1}\right]$$

$$-\frac{1}{2}\lim_{b\to\infty}\left[\underbrace{\frac{1}{e^{b^2}}}_{0}-\frac{1}{e}\right]$$

$$\frac{1}{2e}$$