

Ex. 3

from 8.5

$$\int \frac{x^3 - x + 3}{x^2 + x - 2} dx$$

Handwritten polynomial division:

$$\begin{array}{r} x^2 + x - 2 \overline{) x^3 - x + 3} \\ \underline{x^2 + x - 2} \\ x^3 - x + 3 \\ \underline{x^3 + x^2 - 2x} \\ -x^2 + x + 3 \\ \underline{-x^2 + x - 2} \\ 2x + 1 \end{array}$$

$$\int \frac{x^3 - x + 3}{x^2 + x - 2} dx = \int x - 1 + \frac{2x+1}{x^2+x-2} dx$$

$$\frac{x^2}{2} - x + \int \frac{2x+1}{x^2+x-2} dx$$

Handwritten substitution: $u = x^2 + x - 2$
 $du = 2x + 1 dx$

$$\frac{x^2}{2} - x + \int \frac{1}{u} du$$

Handwritten result: $\ln|x^2+x-2| + C$

$$\int \frac{4x}{e^x} dx$$

Handwritten LATE notation:

$$\begin{array}{l} L \\ \div \\ A \\ \cdot \\ E \end{array} \begin{array}{l} 4x \\ e^{-x} dx \end{array}$$

$$\int 4x e^{-x} dx$$

$$\begin{aligned} uv - \int v du \\ 4x(-e^{-x}) - \int -e^{-x} 4 dx \\ -4xe^{-x} + 4 \int e^{-x} dx \\ -4xe^{-x} + 4(-e^{-x}) + C \end{aligned}$$

$$-\frac{4x}{e^x} - \frac{4}{e^x} + C$$

Handwritten substitution:

$$u = 4x$$

$$du = 4 dx$$

Handwritten substitution:

$$v = -e^{-x}$$

$$dv = e^{-x} dx$$

Handwritten integration:

$$\int e^{-x} dx \quad u = -x$$

$$du = -dx$$

$$- \int e^u du$$

$$-e^u$$

$$-e^{-x}$$

20. $\int_0^1 \frac{5x+8}{x^2+3x+2} dx$ is $\frac{A}{x+2} + \frac{B}{x+1}$

(A) $\ln(8)$ (B) $\ln\left(\frac{27}{2}\right)$ (C) $\ln(18)$ (D) $\ln(288)$ (E) divergent

$$5x+8 = A(x+1) + B(x+2)$$

$$x=-1 \quad 3=B \quad x=-2 \quad -2=-A \quad 2=A$$

$$\int_0^1 \frac{2}{x+2} + \frac{3}{x+1} dx$$

$$2\ln|x+2| + 3\ln|x+1| \Big|_0^1$$

$$\ln|x+2|^2 + \ln|x+1|^3 \Big|_0^1$$

$$\ln(x+2)^2(x+1)^3 \Big|_0^1$$

$$F(1) - F(0)$$

$$\ln(3^2)(2)^3 - \ln(2)^2(1)^3$$

$$\ln 72 - \ln 4$$

$$\ln \frac{72}{4}$$

$$\ln 18$$

$$4 \overline{) \begin{array}{r} 18 \\ 4 \\ 32 \end{array}}$$

20) C

2. $\int 5x(\sqrt{x} - x^2) dx =$

(A) $\frac{15\sqrt{x}}{2} - 15x^2 + C$

(B) $5x - \frac{5x^4}{4} + C$

(C) $2x^{5/2} - \frac{5x^4}{4} + C$

(D) $\frac{25x^{5/2}}{2} - \frac{5x^4}{4} + C$

(E) $\frac{5x^{7/2}}{3} - \frac{5x^6}{6} + C$

$$\int 5x^{3/2} - 5x^3 dx$$

2)

C

1. $\int \frac{x^3 + 5}{x^2} dx =$

(A) $1 - \frac{10}{x^3} + C$

(B) $\frac{3x}{4} + \frac{15}{x^2} + C$

(C) $\frac{x^2}{2} - \frac{5}{x} + C$

(D) $\frac{x^2}{2} - \frac{5}{3x^3} + C$

(E) $-\frac{x^3}{4} - 5 + C$

$$\int \frac{x^3}{x^2} + \frac{5}{x^2} dx$$

$$\int x + 5x^{-2} dx$$

1)

C

Stand and Deliver

Strategies for Finding Limits Algebraically 1.3

1. Try direct substitution
2. If direct substitution fails,
try simplifying or factoring.
3. If direct substitution fails,
and there is a $\sqrt{\quad}$ in the numerator,
rationalize the numerator.
(multiply by the conjugate)

4. L'Hopital's Rule

If $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\infty}{\infty}$

then $\lim_{x \rightarrow c} \frac{f'(x)}{g'(x)}$

Indeterminate form

not quotient rule...

Ex. 1

show both ways...L'Hopitals and simplify, with direct sub

$$\lim_{x \rightarrow -1} \frac{x^2 - x - 2}{x + 1} \quad \frac{1 + 1 - 2}{-1 + 1} = \frac{0}{0}$$

$$\lim_{x \rightarrow -1} \frac{(x-2)(\cancel{x+1})}{\cancel{x+1}}$$

$$\frac{-1-2}{-3}$$

Ex. 1

show both ways...L'Hopitals and simplify, with direct sub

$$\lim_{x \rightarrow -1} \frac{x^2 - x - 2}{x + 1} \quad \frac{1+1-2}{-1+1} = \frac{0}{0}$$

L'Hopital's

$$\lim_{x \rightarrow -1} \frac{2x - 1}{1}$$

$$\frac{2(-1) - 1}{-3}$$

Ex. 2

$$\lim_{x \rightarrow \infty} \frac{\ln x}{2x} \quad \frac{\infty}{\infty}$$

L'Hopital's

$$\lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{2}$$

$$\lim_{x \rightarrow \infty} \frac{1}{2x}$$

$$0$$

Ex. 3

$$\lim_{x \rightarrow \infty} \frac{3x^2}{e^x} \quad \frac{\infty}{\infty}$$

L'Hopital's

$$\lim_{x \rightarrow \infty} \frac{6x}{e^x} \quad \frac{\infty}{\infty}$$

L'Hopital's

$$\lim_{x \rightarrow \infty} \frac{6}{e^x}$$

0

p. 575

Indeterminate Form

| | | |
|------------------|---|-------------------------|
| $\frac{0}{0}$ | | $\frac{\infty}{\infty}$ |
| $0 \cdot \infty$ | 0^0 0*anything=0 anything ⁰ =1 | 1^∞ show =e |
| ∞^0 | | $\infty - \infty$ |

Be careful
these are NOT
indeterminate forms



Which means you can
NOT do L'Hopital's

$$\infty + \infty \rightarrow \infty$$

$$0^{\infty} \rightarrow 0$$

$$-\infty - \infty \rightarrow -\infty$$

$$0^{-\infty} \rightarrow \infty$$



The forms $0/0$, ∞/∞ , $\infty - \infty$, $0 \cdot \infty$, 0^0 , 1^{∞} , and ∞^0 have been identified as *indeterminate*. There are similar forms that you should recognize as “determinate.”

$$\infty + \infty \rightarrow \infty$$

Limit is positive infinity.

$$-\infty - \infty \rightarrow -\infty$$

Limit is negative infinity.

$$0^{\infty} \rightarrow 0$$

Limit is zero.

$$0^{-\infty} \rightarrow \infty$$

Limit is positive infinity.

Ex. 5

rewrite...

$$\lim_{x \rightarrow \infty} e^{-x} \sqrt[3]{x}$$

$$\lim_{x \rightarrow \infty} \frac{x^{1/3}}{e^x} \quad \frac{\infty}{\infty}$$

L'Hopital's

$$\lim_{x \rightarrow \infty} \frac{\frac{1}{3}x^{-2/3}}{e^x}$$

$$\lim_{x \rightarrow \infty} \frac{1}{3x^{2/3}e^x}$$

0

3. $\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x}$ is $\frac{0}{0} = \frac{0}{0}$

(A) -2

(B) 0

(C) 1

(D) 2

(E) nonexistent

L'Hopital's

$$\lim_{x \rightarrow 0} \frac{2x}{\sin x} = \frac{0}{0}$$

L'Hopital

$$\lim_{x \rightarrow 0} \frac{2}{\cos x}$$

2/1

3)

28. Let g be a continuously differentiable function with $\underline{g(1) = 6}$ and $g'(1) = 3$. What is $\lim_{x \rightarrow 1} \frac{\int_1^x g(t) dt}{g(x) - 6}$?
- (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) The limit does not exist.

$$\lim_{x \rightarrow 1} \frac{\int_1^x g(t) dt}{g(x) - 6}$$

$$\lim_{x \rightarrow 1} \frac{\int_1^1 g(t) dt}{g(1) - 6} = \frac{0}{6-6} = \frac{0}{0}$$

28)

$$\lim_{x \rightarrow 1} \frac{\int_1^x g(t) dt}{g(x) - 6}$$

L'Hopital's

$$\frac{\frac{d}{dx} \int_1^x g(t) dt}{g'(x)}$$

$$\lim_{x \rightarrow 1} \frac{g(x)}{g'(x)}$$

$$\frac{g(1)}{g'(1)} = \frac{6}{3} = 2$$