8.5 Partial Fractions

$$\int \frac{1}{x^2-5x+6}dx$$

$$(x-2)(x-3)\left(\frac{1}{x^2-5x+6}\right) = \left(\frac{\cancel{A}\,-1}{x-2} + \frac{\cancel{B}\,1}{x-3}\right)(x-2)(x-3)$$

$(x-2)(x-3)$

$$1 = A(x-3) + B(x-2)$$

Let $x=3$
$1 = B(1)$
$1 = B$

$x=2$
$1 = A(-1)$
$-1 = A$

$$\int \frac{1}{x^2-5x+6}dx = \int \frac{-1}{x-2} + \frac{1}{x-3}\,dx$$

$u = x-2$
$du = dx$

$-\int \frac{1}{u}\,du$

$$-\ln|x-2| + \ln|x-3| + C$$

$$\ln|x-3| - \ln|x-2| + C$$

$$\ln\left|\frac{x-3}{x-2}\right| + C$$

Ex. 1 need partial fractions...

$$\int \frac{1}{x^2-7x+12}\,dx$$

$$\frac{1}{x^2-7x+12} = \frac{A}{(x-3)} + \frac{B}{(x-4)}$$

$$1 = A(x-4) + B(x-3)$$

$x=4$

$1 = B$

$x=3$

$1 = -A$

$-1 = A$

$$\int \frac{-1}{x-3} + \frac{1}{x-4}\,dx$$

$$-\ln|x-3| + \ln|x-4| + c$$

Ex. 2

$$\int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx$$

$x(x^2+2x+1)$

$x(x+1)(x+1)$

$x(x+1)^2$

$$x(x+1)^2\left(\frac{5x^2+20x+6}{x(x+1)^2}\right) = \left(\frac{\overset{6}{\cancel{A}}}{x} + \frac{\overset{-1}{\cancel{B}}}{x+1} + \frac{\overset{9}{\cancel{C}}}{(x+1)^2}\right)x(x+1)^2$$

$$5x^2+20x+6 = A(x+1)^2 + Bx(x+1) + Cx$$

$x=-1$

$5(-1)^2+20(-1)+6 = C(-1)$

$-9 = -C$

$9 = C$

$x=0$

$6 = A$

pick "nice" value for x

$x=1$

$C=9 \quad A=6$

$$5x^2+20x+6 = A(x+1)^2 + Bx(x+1) + Cx$$

$5+20+6 = 6(1+1)^2 + B(1)(2) + 9$

$31 = 24 + 2B + 9$

$-2 = 2B$

$-1 = B$

$$\int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx = \int \frac{6}{x} + \frac{-1}{x+1} + \frac{9}{(x+1)^2}\,dx$$

$$\int \frac{6}{x} - \frac{1}{x+1} + \frac{9}{(x+1)^2}\,dx$$

$$6\ln|x| - \ln|x+1| + 9\int \frac{1}{(x+1)^2}\,dx$$

$u = x+1$

$du = dx$

$$+9\int \frac{1}{u^2}\,du$$

$$+9\int u^{-2}\,du$$

$$\frac{9u^{-1}}{-1}$$

$$6\ln|x| - \ln|x+1| - \frac{9}{x+1} + c$$

$$\ln x^6 - \ln|x+1| - \frac{9}{x+1} + c$$

$$\ln\frac{x^6}{|x+1|} - \frac{9}{x+1} + c$$

AP Question

86. $\int \frac{1\ dx}{(x-1)(x+3)} =$ $\frac{A}{x-1} + \frac{B}{x+3}$

(A) $\frac{1}{4} \ln \left| \frac{x-1}{x+3} \right| + C$

(B) $\frac{1}{4} \ln \left| \frac{x+3}{x-1} \right| + C$

(C) $\frac{1}{2} \ln \left| (x-1)(x+3) \right| + C$

(D) $\frac{1}{2} \ln \left| \frac{2x+2}{(x-1)(x+3)} \right| + C$

(E) $\ln \left| (x-1)(x+3) \right| + C$

$1 = A(x+3) + B(x-1)$

$x=-3$
$1 = -4B$
$-\frac{1}{4} = B$

$x=1$
$1 = 4A$
$\frac{1}{4} = A$

$\int \frac{\frac{1}{4}}{x-1} - \frac{\frac{1}{4}}{x+3}\ dx$

$\frac{1}{4}\int \frac{1}{x-1} - \frac{1}{x+3}\ dx$

$\int \frac{1}{4(x-1)} + \frac{1}{4(x+3)}\ dx$

ans