Warm up

$$\int x \sin x dx$$

L
I
A x
T sinx dx
E

$\int u\,dv = uv - \int v\,du$

$u = \underline{x}$ $v = -\cos x$

$du = dx$ $dv = \underline{\sin x\, dx}$

$x(-\cos x) - \int -\cos x\, dx$

$-x\cos x + \int \cos x\, dx$

$-x\cos x + \sin x + C$

Ex. 4

$$\int x^2 \cos x dx$$

L
I
A x^2
T cosx dx
E

$u = \underline{x^2}$ $v = \sin x$

$du = 2x\,dx$ $dv = \underline{\cos x\, dx}$

$uv - \int v\,du$

$x^2(\sin x) - \int \sin x(2x)dx$

$x^2 \sin x - 2\int x \sin x\, dx$

L
I
A x
T sinxdx
E

From previous ex.

$x^2 \sin x - 2\left[-x\cos x + \sin x + C\right]$

$x^2 \sin x + 2x\cos x - 2\sin x + C$

Tabular Method

In problems involving repeated applications of integration by parts, a tabular method, illustrated in Example 7, can help to organize the work. This method works well for integrals of the form $\int x^n \sin ax\,dx$, $\int x^n \cos ax\,dx$, and $\int x^n e^{ax}\,dx$.

EXAMPLE 7 Using the Tabular Method

Find $\int x^2 \sin 4x\,dx$.

Solution Begin as usual by letting $u = x^2$ and $dv = v'\,dx = \sin 4x\,dx$. Next, create a table consisting of three columns, as shown.

Alternate Signs	*u and Its Derivatives*	*v′ and Its Antiderivatives*
$+$	x^2	$\sin 4x$
$-$	$2x$	$-\frac{1}{4}\cos 4x$
$+$	2	$-\frac{1}{16}\sin 4x$
$-$	0	$\frac{1}{64}\cos 4x$

Differentiate until you obtain 0 as a derivative.

The solution is obtained by adding the signed products of the diagonal entries:

$$\int x^2 \sin 4x\,dx = -\frac{1}{4}x^2\cos 4x + \frac{1}{8}x\sin 4x + \frac{1}{32}\cos 4x + C. \quad ■$$

Ex. 5 stop the madness...

$\int u\,dv = uv - \int v\,du$

L
I
A
T $\cos 2x$
E $e^x\,dx$

$u = \underline{\cos 2x}$ $v = e^x$

$du = -\sin(2x)\cdot 2 = -2\sin 2x$ $dv = \underline{e^x\,dx}$

$\int e^x \cos 2x\,dx = \cos 2x\, e^x - \int e^x(-2\sin 2x)\,dx$

$\int e^x \cos 2x\,dx = \cos 2x\, e^x + 2\left[\int e^x \sin 2x\,dx \atop uv - \int v\,du\right]$

L
I
A
T $\sin 2x$
E $e^x\,dx$

$u = \underline{\sin 2x}$ $v = e^x$

$du = 2\cos 2x\,dx$ $dv = \underline{e^x\,dx}$

$\int e^x \cos 2x\,dx = \cos 2x\, e^x + 2\left[\sin 2x\, e^x - \int e^x(2\cos 2x)\,dx\right]$

$\int e^x \cos 2x\,dx = \cos 2x\, e^x + 2\sin 2x\, e^x - 4\int e^x \cos 2x\,dx$

$+4\int e^x \cos 2x\,dx \qquad\qquad +4\int e^x \cos 2x\,dx$

$\dfrac{5\int e^x \cos 2x\,dx}{5} = \dfrac{\cos 2x\, e^x + 2\sin 2x\, e^x}{5} + C$

$= \dfrac{\cos 2x\, e^x + 2\sin 2x\, e^x}{5} + C$