8.2 Integration by Parts

Objective: You will be able to find an antiderivative using **integration by parts**.

read p. 527

Why do we need integration by parts?

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When is it useful?

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Where did it come from?

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How should we choose dv and u?

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Integration by Parts

In this section you will study an important integration technique called **integration by parts.** This technique can be applied to a wide variety of functions and is particularly useful for integrands involving *products* of algebraic and transcendental functions. For instance, integration by parts works well with integrals such as

$$\int x \ln x \, dx, \quad \int x^2 e^x \, dx, \quad \text{and} \quad \int e^x \sin x \, dx.$$

Integration by parts is based on the formula for the derivative of a product

$$\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$$
$$= uv' + vu'$$

where both u and v are differentiable functions of x. If u' and v' are continuous, you can integrate both sides of this equation to obtain

$$uv = \int uv' \, dx + \int vu' \, dx$$
$$uv = \int u \, dv + \int v \, du.$$

$$uv - \int v\,du = \int u\,dv$$

By rewriting this equation, you obtain the following theorem.

THEOREM 8.1 INTEGRATION BY PARTS

If u and v are functions of x and have continuous derivatives, then

$$\int u\,dv = uv - \int v\,du.$$

This formula expresses the original integral in terms of another integral. Depending on the choices of u and dv, it may be easier to evaluate the second integral than the original one. Because the choices of u and dv are critical in the integration by parts process, the following guidelines are provided.

GUIDELINES FOR INTEGRATION BY PARTS

1. Try letting dv be the most complicated portion of the integrand that fits a basic integration rule. Then u will be the remaining factor(s) of the integrand.
2. Try letting u be the portion of the integrand whose derivative is a function simpler than u. Then dv will be the remaining factor(s) of the integrand.

Note that dv always includes the dx of the original integrand.

Stand and Deliver

Integration by Parts

8.2

$$\int u\,dv = uv - \int v\,du$$

"ultra violet voodoo"

(Higher up determines u)

u = simple dv= most complicated you can integrate	L log I inverse trig A algebraic T trig E exponential (variable in exponent)

Ex. 1

$$\int xe^{2x}dx$$

L
I x
A
T
E $e^{2x}dx$

$u = x \qquad v = \frac{1}{2}e^{2x}$

$du = dx \qquad dv = e^{2x}dx$

$uv - \int v\,du$

$x\left(\frac{1}{2}e^{2x}\right) - \int \frac{1}{2}e^{2x}dx$

$\frac{xe^{2x}}{2} - \frac{1}{2}\int e^{2x}dx$

$\int e^{2x}dx$

$u = 2x$

$du = 2dx$

$\frac{1}{2}du = dx$

$\frac{1}{2}\int e^{u}du$

$\frac{1}{2}e^{2x}$

$\frac{xe^{2x}}{2} - \frac{1}{2}\left(\frac{1}{2}e^{2x}\right) + C$

$\frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + C$

Ex. 2

$$\int x^4 \ln x\,dx$$

L $\ln x$
I
A $x^4\,dx$
T
E

$u = \ln x$ $\quad v = \frac{x^5}{5}$

$du = \frac{1}{x}\,dx$ $\quad dv = x^4\,dx$

$uv - \int v\,du$

$\ln x\left(\frac{x^5}{5}\right) - \int \frac{x^5}{5} \cdot \frac{1}{x}\,dx$

$\frac{x^5 \ln x}{5} - \frac{1}{5}\int x^4\,dx$

$\frac{x^5 \ln x}{5} - \frac{1}{5}\left(\frac{x^5}{5}\right) + C$

$\frac{x^5 \ln x}{5} - \frac{x^5}{25} + C$

Ex. 3

$$\int 4 \arccos x\,dx$$

L — arccos x
I
A — 4 dx
T
E

$u = \arccos x \qquad v = 4x$

$du = \dfrac{-1}{\sqrt{1-x^2}}dx \qquad dv = 4\,dx$

$uv - \int v\,du$

$\arccos x(4x) - \int 4x\left(-\dfrac{1}{\sqrt{1-x^2}}\right)dx$

$4x\arccos x + 4\left[\int \dfrac{x}{\sqrt{1-x^2}}dx\right]$

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

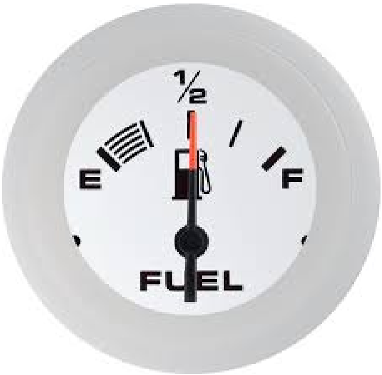
$4x\arccos x + 4\left[-\frac{1}{2}\int u^{-1/2}\,du\right]$

$\left[-\frac{1}{2}\,2 \cdot u^{\frac{1}{2}}\right]$

$4x\arccos x + 4\left[-(1-x^2)^{1/2}\right]$

$4x\arccos x - 4\sqrt{1-x^2} + C$

How are we doing?



Did we meet our goal for the day?

Objective: You will be able to find an antiderivative using integration by parts.