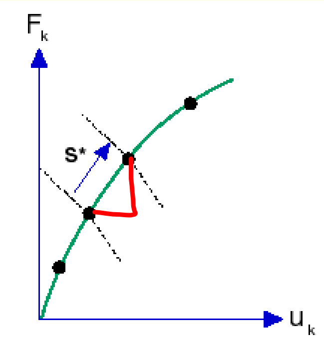
1. Area WS
2. Volume Disk/Washer
3. FRQ packet
4. AP Review packet
5. Book Review

7.4 Arc Length

Stand and Deliver

Arc Length

7.4



$$S = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

$$S = \int_c^d \sqrt{1 + (g'(y))^2}\, dy$$

Justification for the arc length formula

https://www.khanacademy.org/math/ap-calculus-ab/area-and-arc-length-ab/arc-length-ab/v/arc-length-formula

mathispower4u.com
calc II

"More Applications of Integration"
arc length part I

nice explanation of how formula is derived
next slide shows a simplified version

https://www.youtube.com/watch?v=seoFxrNL85c

(first 3:50 explains where formula comes from)

Ex. 2

Find the arc length: $y = 2x^{3/2} + 3$ [0, 9]

$S = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$

$y = 2x^{3/2} + 3$

$y' = 3x^{1/2}$

$(y')^2 = 9x$

$\int_0^9 \sqrt{1 + 9x}\, dx$

$u = 1 + 9x$

$du = 9\,dx$

$\frac{1}{9} du = dx$

$\frac{1}{9} \int u^{1/2}\, du$

$\frac{1}{9} \cdot \frac{2}{3} \cdot u^{3/2}$

$\frac{2}{27} \left[(1 + 9x)^{3/2} \right]_0^9$

$F(9) - F(0)$

$\frac{2}{27} \left[82^{3/2} - 1^{3/2} \right]$

$\frac{2}{27} \left[82^{3/2} - 1 \right]$

Ex. 3

Find the arc length- use calculator:

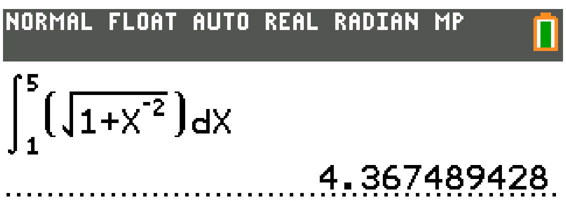
$y = \ln x$ [1, 5]

$y = \ln x$

$y' = \frac{1}{x}$

$(y')^2 = \frac{1}{x^2}$

$\int_1^5 \sqrt{1 + \left(\frac{1}{x^2}\right)}\, dx$



Ex. 4

Find the arc length- no calculator:

$y = \frac{1}{3}(x^2+2)^{3/2}$; $0 \le x \le 1$

$y' = \frac{1}{2}(x^2+2)^{1/2}(2x)$

$y' = x(x^2+2)^{1/2}$

$(y')^2 = x^2(x^2+2)$

$= x^4+2x^2$

$\int_0^1 \sqrt{1+x^4+2x^2}\,dx$

$\int_0^1 \sqrt{x^4+2x^2+1}\,dx$

$\int_0^1 \sqrt{(x^2+1)^2}\,dx$

$\int_0^1 x^2+1\,dx$

$\left.\frac{x^3}{3}+x\right]_0^1$

$F(1)-F(0)$

$\frac{1}{3}+1-0$

$1\frac{1}{3}$

$\frac{4}{3}$

2011 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

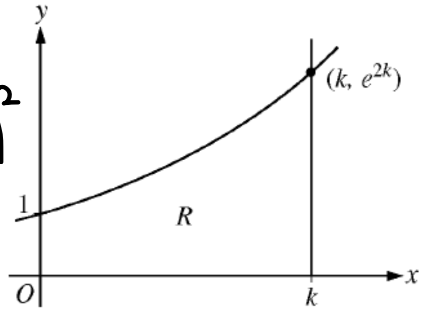
CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

$f(x)=e^{2x}$

$f'(x)=(2e^{2x})^2$

$(f'(x))^2=4e^{4x}$



3. Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of f, the coordinate axes, and the vertical line $x = k$, where $k > 0$. The region R is shown in the figure above.

(a) Write, but do not evaluate, an expression involving an integral that gives the perimeter of R in terms of k.

$1+k+e^{2k}+\int_0^k \sqrt{1+4e^{4x}}\,dx$

(a) $f'(x) = 2e^{2x}$

Perimeter $= 1 + k + e^{2k} + \int_0^k \sqrt{1 + \left(2e^{2x}\right)^2}\, dx$

$3 : \begin{cases} 1 : f'(x) \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

Surface Area

read p. 482-483
follow examples on p. 484

DEFINITION OF THE AREA OF A SURFACE OF REVOLUTION

Let $y = f(x)$ have a continuous derivative on the interval $[a, b]$. The area S of the surface of revolution formed by revolving the graph of f about a horizontal or vertical axis is

$$S = 2\pi \int_a^b r(x)\sqrt{1 + [f'(x)]^2}\, dx$$

y is a function of x.

where $r(x)$ is the distance between the graph of f and the axis of revolution. If $x = g(y)$ on the interval $[c, d]$, then the surface area is

$$S = 2\pi \int_c^d r(y)\sqrt{1 + [g'(y)]^2}\, dy$$

x is a function of y.

where $r(y)$ is the distance between the graph of g and the axis of revolution.

https://www.youtube.com/watch?v=4XLq-BWK5NY